hep-ph/0406075

Discriminating neutrino mass models using Type II seesaw formula

N.Nimai Singh[a,c,1], Mahadev Patgiri[b] and Mrinal Kumar Das[c]

[a] *International Centre for Theoretical Physics, Strada Costiera 11, 31014 Trieste, Italy*
[b] *Department of Physics, Cotton College, Guwahati-781001, India*
[c] *Department of Physics, Gauhati University, Guwahati-781014, India*

Abstract

In this paper we propose a kind of natural selection which can discriminate the three possible neutrino mass models, namely the degenerate, inverted hierarchical and normal hierachical models, using the framework of Type II seesaw formula. We arrive at a conclusion that the inverted hierarchical model appears to be most favourable whereas the normal hierarchical model follows next to it. The degenerate model is found to be most unfavourable. The neutrino mass matrices which are obtained using the usual canonical seesaw formula (Type I), and which also give almost good predictions of neutrino masses and mixings consistent with the latest neutrino oscillation data, are re-examined in the light of non-canonical seesaw formula (Type II). We then estimate a parameter γ which represents the minimum degree of suppression of the extra term arising from the left-handed Higgs triplet, so as to restore the good predictions on neutrino masses and mixings already had in Type I seesaw model.

[1]Regular Associate of ICTP,
e-mail address: nsingh@ictp.trieste.it; nimai03@yahoo.com

1 Introduction

Recent neutrino oscillation experiments[1] which provide important informations on the nature of neutrino masses and mixings, have strengthened our understanding of neutrino oscillation[2,3]. However we are still far from a complete understanding of neutrino physics. One of them is the pattern of the neutrino mass eigenvalues, though some reactor experiments are trying to understand it. For future reference we summarise here[1] the most recent results of the three-flavour neutrino oscillation parameters from global data including solar[4], atmospheric[5], reactor (KamLAND[6] and CHOOZ[7]) and accelerator (K2K [8]):

parameter	best fit	3σ level
$\triangle m^2_{21}[10^{-5}eV^2]$	6.9	$5.4 - 9.4$
$\triangle m^2_{23}[10^{-3}eV^2]$	2.3	$1.1 - 3.4$
$\sin^2\theta_{12}$	0.3	$0.23 - 0.39$
$\sin^2\theta_{23}$	0.52	$0.32 - 0.70$
$\sin^2\theta_{13}$	0.005	≤ 0.061

As far as the LSND result[9] is concerned, it is finding difficulty to reconcile with the rest of the global data, and a confirmation of the LSND signal by the MiniBooNE experiment[10] would be very desirable. There are also some complementary information from other sources. The recent analysis of the WMAP collaboration[11] gives the bound $\sum_i |m_i| < 0.69$eV (at 95% C.L.) (more conservative analysis[12] gives $\sum_i |m_i| < 1.01$eV). The bound from the $0\nu\beta\beta$-decay experiment is $|m_{ee}| < 0.2$eV (a more conservative analysis gives $|m_{ee}| < (0.3 - 0.5)$eV)[13,14]. However the value of the $|m_{ee}|$ from the recent claim[15] for the discovery of the $0\nu\beta\beta$ process at 4.2σ level, is $|m_{ee}| \sim (0.2 - 0.6)$eV (more conservative estimate involving nuclear mass is $|m_{ee}| \sim (0.1 - 0.9)$eV).

Since the above data on solar and atmospheric neutrino oscillation experiments, gives only the mass square differences, we usually have three models[2] of neutrino mass levels[16]:
Degenerate (Type [I]): $m_1 \simeq m_2 \simeq m_3.$,

[2]In order to avoid possible confusion in nomenclature, Types of neutrino mass models are denoted inside the square bracket whereas Types of seesaw formula are expressed without square bracket.

Inverted hierarchical (Type [II]): $m_1 \simeq m_2 >> m_3$ with $\triangle m^2_{23} = m^2_3 - m^2_2 < 0$ and $m_{1,2} \simeq \sqrt{\triangle m^2_{23}} \simeq (0.03 - 0.07)$eV; and
Normal hierarchical (Type [III]): $m_1 << m_2 << m_3$, and $\triangle m^2_{23} = m^2_3 - m^2_2 > 0$; and $m_3 \simeq \sqrt{\triangle m^2_{23}} \simeq (0.03 - 0.07)$ eV, and $m_2 \simeq 0.008$eV.
(Appendix A presents a list of the zeroth-order left-handed Majorana mass matrices which can explain the above three patterns of neutrino masses). The result of $0\nu\beta\beta$ decay experiment, if confirmed, would be able to rule out Type [II] and Type [III] neutrino mass models straight, and points to Type [I] or to models with more than three neutrinos[3]. Again, the WMAP limit (at least for three degenerate neutrinos), $|m| < 0.23$eV also would rule out Type [I] neutrino model, or at least it could lower the parameter space for the degenerate model[3]. It also gives further constraint on $|m_{ee}|$. However a final choice among these three models is a difficult task. At the moment we are in a very confusing state. The present paper is a modest attempt from a theoretical point of view to discriminate the neutrino mass models using the Type II seesaw formula (non-canonical seesaw formula) for neutrino masses.

The paper is organised as follows. In section 2, we outline the main points of the Type II seesaw formula and a criteria for a natural selection which helps to discriminate the neutrino mass models. We carry out numerical computations in section 3 and present our main results. Section 4 concludes with a summary and discussions.

2 Type II see-saw formula and neutrino mass matrix

The canonical seesaw mechanism (generally known as Type I seesaw formula)[17] is the simplest and most appealing mechanism for generating small neutrino masses and lepton mixings. There is also another type of seesaw formula (known as Type II seesaw formula)[18] where a left-handed Higgs triplet Δ_L picks up a vacuum expectation value (vev) in the left-right symmetric GUT models such as SO(10). This is expressible as

$$m_{LL} = m^{II}_{LL} + m^{I}_{LL}. \qquad (1)$$

where the usual Type I seesaw formula is given by the expression,

$$m^{I}_{LL} = -m_{LR} M^{-1}_{RR} m^T_{LR} \qquad (2)$$

Here m_{LR} is the Dirac neutrino mass matrix in the left-right convention and the right-handed Majorana neutrino mass matrix $M_{RR} = v_R f_R$ with v_R being the vacuum expectation value (vev) of the Higgs fields imparting mass to the right-hand neutrinos and f_R is the Yukawa coupling matrix. The second term m_{LL}^{II} is due to the $SU(2)_L$ Higgs triplet, which can arise, for instance, in a large class of $SO(10)$ models in which the $B-L$ symmetry is broken by a 126 Higgs field[19,20]. In the usual left-right symmetric theories, m_{LL}^{II} and M_{RR} are proportional to the vacuum expectation values (vevs) of the electrically neutral components of scalar Higgs triplets, i.e., $m_{LL}^{II} = f_L v_L$ and $M_{RR} = f_R v_R$, where $v_{L,R}$ denotes the vevs and $f_{L,R}$ is a symmetric 3×3 matrix. By acquiring the vev v_R, breaking of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ to $SU(2)_L \times U(1)_Y$ is achieved. The left-right symmetry demands the presence of both m_{LL}^{II} and M_{RR}, and in addition, it holds $f_R = f_L = f$. The induced vev for the left-handed triplet v_L is given by $v_L = \gamma M_W^2 / v_R$, where the weak scale $M_W \sim 82$GeV such that $|v_L| << M_W << |v_R|$[21][3]. In general γ is a function of various couplings, and without fine tuning γ is expected to be of the order unity. Type II seesaw formula in Eq.(1) can now be expressed as

$$m_{LL} = \gamma (M_W / v_R)^2 M_{RR} - m_{LR} M_{RR}^{-1} m_{LR}^T \tag{3}$$

In the light of the above Type II seesaw formula Eq.(3), the neutrino mass matrices, m_{LL} in literature, are constructed in view of the following three assumptions: (a) m_{LL}^{II} is dominant over m_{LL}^{I}, and (b) both terms are contributing with comparable amounts, (c) m_{LL}^{I} is dominant over m_{LL}^{II}. In recent times case(a) has gathered momentum because in certain $SO(10)$ models, large atmospheric neutrino mixing and $b-\tau$ unification are the natural outcomes of this dominance[20,22,23]. In some models this leads to degenerate model [24] which imparts bimaximal mixings, as well as extra contribution to leptogenesis[24,25,26]. However all these cases are not completely free from certain assummptions as well as ambiguities.

It can be stressed that these two terms m_{LL}^{I} and m_{LL}^{II} in Eq.(1), are not completely independent. The term m_{LL}^{II} is heavily constrained through v_R as seen in Eq.(3). Usually the value of v_R is fixed through the definition $M_{RR} = v_R f$ present in the canonical term m_{LL}^{I}. There is no ambiguity

[3]In some papers[19] $v_u \sim 250$GeV is taken in place of M_W. We prefer here to take $\sim$ 82GeV as it is nearer to our input value of either $m_t = 82.43$GeV or $m_\tau \tan\beta = 1.3 \times 40$GeV in the text. In this way both the terms of the Type II seesaw formula, have almost same value of weak scale. However taking different values does not alter the conclusion of our analysis

in the definition of v_R with the first term, and it also does not affect m_{LL}^I as long as M_{RR} is taken as a whole in the expression. However it severly affects the second term m_{LL}^{II} where v_R is entered alone, and different choices of v_R in M_{RR} would lead to different values of m_{LL}^{II}. This ambiguity is seen in the literature where different choices of v_R are made according to the convenience[19,21,22,25,28]. However, in the present paper we shall always take v_R as the heaviest right-handed Majorana neutrino mass eigenvalue obtained after the diagonalisation of the mass matrix M_{RR}. Once we adopt this convention, there is little freedom for the second term m_{LL}^{II} in Eq.(3) to have arbitrary value of v_R. We also assume that the $SU(2)_R$ gauge symmetry breaking scale v_R is the same as the scale of the breakdown of parity [20][4].

The present work is carried out in the line of case (b) and (c) discussed above, but the choice of which term is dominant over other, is not arbitrary any more. We carry out a complete analysis of the three models of neutrino mass matrices (See Appendix B for the expressions of M_{RR} and m_{LL}) where the (already acquired) good predictions of neutrino masses and mixings in the canonical term m_{LL}^I is subsequently spoiled by the presence of second term m_{LL}^{II} when $\gamma = 1$ in m_{LL}. We make a search programme for finding the values of the minimum departure of γ from of the canonical value of one, in which the good predictions of neutrino masses and mixing parameters can be restored in m_{LL}. We propose here a bold hypothesis which acts as a sort of " natural selection" for the survival of the neutrino mass models which enjoy the least value of deviation of γ from unity. In other words, the value of γ is enough just to suppress the perturbation effect arising from Type II seesaw formula. Nearer the value of γ to one, better the chance for the survival of the model in question. Thus the parameter γ is an important parameter for the proposed natural selection of the neutrino mass models.

The above criteria for natural selection imposes certain constraints on the neutrino mass models which one can obtain in the following way, at least for the heaviest neutrino mass eigenvalue (without considering mixings). If the neutrino masses are solely determined from the second term of Eq.(3), then the first term must be less than the certain order which is dictated by the particular pattern of neutrino mass spectrum. In this view, the largest contribution of neutrino mass from the first term must be less than about 0.05eV for both normal hierarchical and inverted hierarchical models; and about 0.5eV for degenerate model as the data suggests[1]. Thus we have the

[4]See Ref.[37] for further discussion on the choice of v_R

bound for the natural selection:

$$m_{LL}^{I} > v_L f \tag{4}$$

Denoting the heaviest right-handed neutrino mass as v_R and taking $M_W \sim$ 82GeV[21] in the expression of v_L, the following lower bounds on v_R for the natural selection are obtained:

For normal hierchical and inverted hierarchical model:

$$v_R > \gamma 1.345 \times 10^{14} GeV \tag{5}$$

For degenerate model:

$$v_R > \gamma 1.345 \times 10^{13} GeV \tag{6}$$

The above bounds just indicate the approximate measure of the degree of natural selection, but a fuller analysis will take both the terms of the Type II seesaw formula in the 3×3 matrix form. This will gives all the three mass eigenvalues as well as mixing angles. This numerical analysis will be carried out in the next section. It is clear from Eqs.(5) and (6) that any amount of arbitrariness in fixing the value of v_R in M_{RR} will distort the conclusion.

3 Numerical calculations and results

For a full numerical analysis we refer to our earlier papers[27] where we performed the investigations on the origin of neutrino masses and mixings which can accomodate LMA MSW solution for solar neutrino anomaly and the solution of atmospheric neutrino problem within the framework of Type I seesaw formula. Normal hierarchical, inverted hierarchical and quasi-degenerate neutrino mass models were constructed from the nonzero textures of the right-handed Majorana mass matrix M_{RR} along with diagonal form of m_{LR} being taken as either the charged lepton mass matrix (case i)[28] or the up-quark mass matrix (case ii)[27]. However, a general form of the Dirac neutrino mass matrix is given by :

$$m_{LR} = \begin{pmatrix} \lambda^m & 0 & 0 \\ 0 & \lambda^n & 0 \\ 0 & 0 & 1 \end{pmatrix} m_f, \tag{7}$$

where m_f corresponds to $(m_\tau \tan\beta)$ for $(m,n) = (6,2)$ in case of charged lepton (**case i**) and m_t for $(m,n) = (8,4)$ in case of up-quarks (**case ii**). The value of the parameter λ is taken as 0.22. Here the assumption is that neutrino mass mixings can arise from the texture of right handed neutrino mass matrix only through the interplay of seesaw mechanism[29]. This can be understood from the following operation[30,31] where M_{RR} can be transformed in the basis in which m_{LR} is approximately diagonal[5]. Using the diagonalisation relation, $m_{LR}^{diag} = U_L m_{LR} U_R^\dagger$, we have,

$$m_{LL}^I = -m_{LR} M_R^{-1} m_{LR}^T$$

$$= -U_L^+ m_{LR}^{diag} U_R M_R^{-1} U_R^T m_{LR}^{diag} U_L^*$$

$$= -U_L^+ m_{LR}^{diag} M_{RR}^{-1} m_{LR}^{diag} U_L^*$$

$$\simeq -m_{LR}^{diag} M_{RR}^{-1} m_{LR}^{diag}$$

where $U_L m_{LL}^I U_L^T \simeq m_{LL}^I$ by considering a simple assumption, $U_L \simeq 1$. Since the Dirac neutrino mass matrices are hierarchical in nature and the CKM mixing angles of the quark sector are relatively small. In such situation U_L slightly deviates from 1, i.e., $U_L \simeq V_{CKM}$, and it hardly affects the numerical accuracy[30] for practical purposes. Here M_{RR} is the new RH matrix defined in the basis of diagonal m_{LR} matrix. We thus express M_{RR} in the most general form as its origin is quite different from those of the Dirac mass matrices in an underlying grand unified theory. As usual the neutrino mass eigenvalues and neutrino mixing matrix (MNS) are obtained through the diagonalization of m_{LL},

$$m_{LL}^{diag} = V_{\nu L} m_{LL} V_{\nu L}^T = Diag(m_1, m_2, m_3),$$

and the neutrino mixing angles are extracted from the MNS lepton mixing matrix defined by $V_{MNS} = V_{\nu L}^\dagger$.

[5]This is also true for any diagonal m_{LR} with any arbitrary pair of (m,n). A corresponding M_{RR} can be found out in principle

Normal hierarchical model (Type [III]):

We then perform a detailed numerical analysis to search for the parameter γ which measures the perturbation effects arising from the Type II seesaw term. As a simplest example, we take up the case for the normal hierarchical model (Type [III]) while the expressions for other models are relegated to Appendix B. Using the general expression for m_{LR} given in Eq.(7) and the following texture for M_{RR} [27]:

$$M_{RR} = \begin{pmatrix} \lambda^{2m-1} & \lambda^{m+n-1} & \lambda^{m-1} \\ \lambda^{m+n-1} & \lambda^{m+n-2} & 0 \\ \lambda^{m-1} & 0 & 1 \end{pmatrix} v_R, \tag{8}$$

we get the neutrino mass matrix of the Type [III] through Eq.(2),

$$-m_{LL}^{I} = \begin{pmatrix} -\lambda^4 & \lambda & \lambda^3 \\ \lambda & 1-\lambda & -1 \\ \lambda^3 & -1 & 1-\lambda^3 \end{pmatrix} \times 0.03eV \tag{9}$$

Here we have fixed the value of $v_R = 8.92 \times 10^{13}$GeV for case (i), taking (m, n) as $(6, 2)$ and the input values $m_\tau = 1.292$GeV, and $\tan\beta = 40$. The diagonalization of M_{RR} gives the three corresponding RH Majorana neutrino masses $M_{RR}^{diag} = (-4.8555 \times 10^8, 1.058 \times 10^{10}, 8.92 \times 10^{13})$GeV. As already stated, the mass matrix in Eq.(9) predicts correct neutrino mass parameters and mixing angles consistent with recent data[27]:

$m_{LL}^{diag} = (0.0033552, 0.0073575, 0.057012)$eV, leading to
$\triangle m_{21}^2 = 4.29 \times 10^{-5}$eV, $\triangle m_{23}^2 = 3.20 \times 10^{-3}$eV, $\alpha = \triangle m_{21}^2 / \triangle m_{23}^2 = 0.0134$.
$\sin\theta_{12} = 0.5838$, $\sin\theta_{23} = 0.6564$, $\sin\theta_{13} = 0.074$.

In the next step we take the additional contribution of the second term $m_{LL}^{II} = \gamma(M_W/v_R)^2 M_{RR}$ in Type II seesaw formula in Eq.(3). When $\gamma = 1$, all the good predictions of neutrino masses and mixings already had in m_{LL}^{I}, are then spoiled. The value of γ for the "least deviation from canonical value of one", which could restore the good predictions in m_{LL}, is again obtained through a search programme. The predictions are: $\gamma \simeq 0.1$, $m_{LL}^{diag} = (-0.0021353, 0.0095481, -0.0534155)$eV, leading to
$\triangle m_{21}^2 = 8.66 \times 10^{-5}$eV, $\triangle m_{23}^2 = 2.76 \times 10^{-3}$eV, $\alpha = \triangle m_{21}^2 / \triangle m_{23}^2 = 0.0314$.

The corresponding MNS mixing matrix which diagonalizes m_{LL} is obtained as

$$V_{MNS} = \begin{pmatrix} 0.879342 & 0.468733 & -0.083946 \\ 0.275275 & -0.644212 & -0.713593 \\ 0.388564 & -0.604384 & 0.695513 \end{pmatrix} \tag{10}$$

leading to $\sin^2 2\theta_{12} = 0.6796$, $\tan^2\theta_{12} = 0.284 < 1$, $\sin^2 2\theta_{23} = 0.98531$, $\sin\theta_{13} = 0.084$.

Here we have given solar mixing angle in term of $tan^2\theta_{12}$ to check whether it falls in the "light side", $\tan^2\theta_{12} < 1$, for the usual sign convention $\triangle m^2_{21} = m^2_2 - m^2_1 > 0$, [32, 33]. For the case (ii) when $(m, n) = (8, 4)$ in Eq.(7), we take the input value $m_t = 82.43$GeV at the high scale. We have again the final predictions from m_{LL}: $\gamma \simeq 0.1$, and $M^{diag}_{RR} = (-2.891 \times 10^6, 6.299 \times 10^7, 2.267 \times 10^{14})$GeV,
$\triangle m^2_{21} = 5.81 \times 10^{-5}$eV, $\triangle m^2_{23} = 3.02 \times 10^{-3}$eV, $\alpha = \triangle m^2_{21}/\triangle m^2_{23} = 0.0192$, $\sin^2 2\theta_{12} = 0.8233$, $\tan^2\theta_{12} = 0.42 < 1$, $\sin^2 2\theta_{23} = 0.9862$, $\sin\theta_{13} = 0.077$.

We also calculate the mass parameter $|m_{ee}|$ measured in the $0\nu\beta\beta$ decay experiment using the expression[3]

$$|m_{ee}| = |(1 - \sin^2\theta_{13})(m_1\cos^2\theta_{12} + m_2\sin^2\theta_{12}) + m_3 e^{2i\phi}\sin^2\theta_{13}| \tag{11}$$

For degenerate (Type [I]) and inverted hierarchy (Type [II]) model, it reduces to (for $\sin^2\theta_{13} \simeq 0$)

$$|m_{ee}| \sim |m|(\cos^2\theta_{12} \pm \sin^2\theta_{12}) \tag{12}$$

In case of normal hierarchy (Type [III]), we have

$$|m_{ee}| \sim |m_2\sin^2\theta_{12} \pm m_3\sin^2\theta_{13}| \tag{13}$$

For the example discussed above (normal hierarchy for case (i)), we obtain $|m_{ee}| = 0.0017$.

In **Appendix B** we list all the m^I_{LL} along with the corresponding M_{RR} textures for Degenerate (Type [I (A,B,C)]) and Inverted hierarchy (Type [II(A,B)])[27]. We repeat the same procedure described above for all these cases and find out the corresponding values of γ.

We present here the main results of the analysis. We calculate RH neutrino masses in Table-1 for both cases (i) and (ii). The heaviest RH Majorana

eigenvalue is taken as v_R scale for calculation of m_{LL}^{II}. It is interesting to see in Table-1 that only Type [II(A,B)] satisfie the bounds given in Eqs.(5) and (6) when $\gamma = 1$. This roughly implies that Inverted hierarchical model is the best choice for natural selection though a fuller analysis needs the matrix form when all terms are present. Our main results on neutrino masses and mixings are presented in Table-2 and Table-3. One particular important parameter is the predicted values of γ. Table-4 presents the mass parameter $|m_{ee}|$ and α for both cases (i) and (ii).

From Table-2 and -3, we wish to make a conclusion that Inverted hierarchy model (Type [II]) having $\gamma = 1$ is the most stable under the presence of $SU(2)_L$ triplet term m_{LL}^{II} in the Type [II] seesaw formula. On such ground we can discriminate other models in favour of it. Next to Inverted hierarchy is the normal hierarchy model (Type [III]) with $\gamma = 0.1$. In the present analysis the degenerate model (Type [I]) is not favourable as it predicts $\gamma \sim 10^{-4}$. Again, in Inverted hierarchy model we have two Types - [IIA] and [IIB]. Type [IIA] predicts slightly lesser solar mixing angle and exactly zero CHOOZ angle without any fine tunning such as contribution from charged lepton etc., compared to those of Type [IIB] where we have maximal solar mixing. The present analysis has limitation to further discriminate either of these two.

As a routine calculation we check the stability of these models under radiative corrections in MSSM for both neutrino mass splittings and mixing angles. For large $\tan\beta = 55$ where the effect of radiative correction is relatively large, only two models namely, Inverted hierarchy [38] of Type [IIB] and Normal hierarchy [39] of Type [III] are found stable under radiative corrections. Following this result, the inverted hierarchy of Type [IIA] is less favourable than its counterpart Type [IIB].

Table-1: The three right-handed Majorana neutrino masses for both case (i) and case (ii) in three pattern of neutrino mass models. The $B-L$ symmetry breaking scale v_R is taken as the mass of the heaviest right-handed Majorana neutrino in the calculation.

Type	Case(i):$\vert M_{RR}^{diag}\vert$GeV	Case(ii):$\vert M_{RR}^{diag}\vert$GeV
[IA]	6.82×10^{9}, 5.51×10^{9}, 5.04×10^{12};	6.34×10^{7}, 1.94×10^{8}, 8.55×10^{12}
[IB]	1.50×10^{5}, 2.72×10^{10}, 1.16×10^{13};	5.17×10^{2}, 9.37×10^{7}, 1.7×10^{13}
[IC]	1.3×10^{5}, 4.61×10^{11}, 5.06×10^{11};	5.17×10^{2}, 1.89×10^{10}, 8.5×10^{10}
[IIA]	1.19×10^{6}, 4.32×10^{11}, 4.63×10^{17};	4.1×10^{3}, 1.49×10^{8}, 6.8×10^{17}
[IIB]	2.97×10^{8}, 2.97×10^{8},1.16×10^{16};	1.74×10^{6},1.74×10^{6},2.89×10^{16}
[III]	4.86×10^{8}, 1.06×10^{10}, 8.92×10^{13};	2.89×10^{6}, 6.3×10^{7}, 2.27×10^{14}

Table-2:Predicted values of γ and its corresponding solar and atmospheric mass-squared differences and mixing parameters from m_{LL} using the values of v_R from Table 1 for case (i).

Type	γ	$\triangle m^2_{21}[10^{-5}eV^2]$	$\triangle m^2_{23}[10^{-3}eV^2]$	$\tan^2\theta_{12}$	$\sin^2_2\theta_{23}$	$\sin\theta_{13}$
[IA]	10^{-4}	8.56	2.76	1.013	0.993	0.03
[IB]	10^{-3}	3.97	2.48	0.278	1.00	0.0
[IC]	10^{-3}	3.65	2.46	2.855	1.00	0.0
[IIA]	1	3.39	2.45	0.282	0.999	0.0
[IIB]	1	10.7	4.91	0.978	1.00	0.004
[III]	0.1	8.66	2.76	0.284	0.985	0.084

Table-3:Predicted values of γ and its corresponding solar and atmospheric mass-squared differences, and mixing parameters from m_{LL} using the values of v_R from Table-1 for case (ii)

Type	γ	$\triangle m^2_{21}[10^{-5}eV^2]$	$\triangle m^2_{23}[10^{-3}eV^2]$	$\tan^2\theta_{12}$	$\sin^2_2\theta_{23}$	$\sin\theta_{13}$
[IA]	10^{-4}	9.04	2.76	1.012	0.994	0.033
[IB]	10^{-3}	3.97	2.47	0.268	1.00	0.0
[IC]	10^{-4}	3.33	2.49	1.760	0.999	0.0
[IIA]	1	3.39	2.47	0.289	0.999	0.0
[IIB]	1	9.05	5.04	0.991	0.999	0.002
[III]	0.1	5.81	3.02	0.42	0.986	0.078

Table-4: Predicted values of the $0\nu\beta\beta$ decay mass parameter $|m_{ee}|$ and α for both cases (i) and (ii) from m_{LL} using the values of parameters in Table 1-3.

Type	Case(i)		Case (ii)	
	$\|m_{ee}\|$	α	$\|m_{ee}\|$	α
[IA]	0.084	0.0311	0.084	0.0328
[IB]	0.3968	0.0160	0.3964	0.016
[IC]	0.3968	0.0148	0.3968	0.0134
[IIA]	0.05	0.0139	0.0501	0.0137
[IIB]	0.0	0.022	0.0	0.018
[III]	0.0	0.0314	0.0	0.0192

4 Summary and Discussion

We summarise the main points of this work. We can generate in principle the three neutrino mass matrices namely, degenerate (Type [I(A,B,C)]), inverted hierarchical (Type [II (A,B)]) and normal hierarchical (Type [III]) models, by taking the diagonal form of the Dirac neutrino mass matrix and a non-diagonal form of the right-handed Majorana mass matrix in the canonical seesaw formula (Type I). We then examine whether these good predictions are spoiled or not in the presence of the left-handed Higgs triplet in Type II seesaw formula; and if so, we find out the least minimum perturbation for retaining good predictions which are previously obtained. We propose a kind of natural selection of the neutrino mass models which have "least perturbation" arising from Type II seesaw term, in order to retain the good predictions already acquired. Under such hypothesis we arrive at the conclusion that inverted hierachical model is the most favourable one in nature. Next to it is the normal hierarchy. Degenerate models are badly spoiled by the perturbation in Type II seesaw formula, and therefore it is not favoured

by the natural selection. Our conclusion also nearly agrees with the calculations using the mass matrices m_{LR} and M_{RR} predicted by other authors in SO(10) models[34,35]. It can be stressed that the method adopted here is also applicable to any neutrino mass matrix obtained using a general non-diagonal texture of Dirac mass matrix.

A few comments are in order. Within the Inverted hierarchical model itself, we are having two varieties: Type [IIA] with mass egenvalues $(m_1, m_2, 0)$ and Type [IIB] with mass eigenvalues $(m_1, -m_2, 0)$. The present analysis could not distingush which one is more favourable as both predict the same $\gamma = 1$ which measures the degree of perturbation arising from Type II seesaw formula. This means that for Inverted model, Type I seesaw term donimates over Type II seesaw term without any fine tuning. However within the inverted hierarchy model, Type [IIA] model predicts slightly lesser solar mixing, $\tan^2\theta_{12} = 0.282$ and $\sin\theta_{13} \sim 0.0$ and $|m_{ee}| \sim 0.05$eV without any fine tunning. Type [IIB] predicts maximal solar mixing $\tan^2\theta_{12} = 0.99$, small CHOOZ angle $\sin\theta_{13} \simeq 0.004$, and $|m_{ee}| \sim 0$. This requires some other mechanisms to tone down the solar angle at the cost of increasing $\sin\theta_{13}$ near the experimental bound. For example, taking charged lepton contribution, one can have in this case $0.66 \geq \tan^2\theta_{12} \geq 0.49$, corresponding to $0.10 \leq |V_{e3}| \leq 0.17$ and $0.014eV \leq |m_{ee}| \leq 0.024eV$ [33]. If we use the lower bound on $|m_{ee}| > 0.013$ eV derived from the SNO data (with salt run)[36], Type [IIB] nearly survives. Precise measurement of $\sin\theta_{13}$ may help to distinguish these two kinds. This can be tested in the future long baseline experiments[36]. As a remark we also point out that unlike Type [IIB] [38], Type [IIA] will be unstable under quantum radiative corrections in MSSM [33]. As emphasised before, the present analysis is based on the hypothesis that those models of neutrinos where the canonical seesaw term is dominant over the perturbative term arising from Type II seesaw, are favourable under natural selection. The present work is a modest attempt to understand the correct model of neutrino mass pattern.

Acknowledgements

One of us (N.N.S.) would like to thank Prof.Goran Senjanovic for useful discussion and Professor Randjbar-Daemi, Head of the High Energy Group, International Centre for Theoretical Physics, Trieste, Italy, for kind hospitality at ICTP during my visit under Regular associateship scheme.

Appendix A

We list here for ready reference[16], the zeroth-order left-handed Majorana neutrino mass matrices with texture zeros, m_{LL}, corresponding to three models of neutrinos, viz., degenerate (Type [I]), inverted hierarchical (Type [II]) and normal hierarchical (Type [III]). These mass matrices are competible with the LMA MSW solution as well as maximal atmospheric mixings.

Type	m_{LL}	m_{LL}^{diag}
[IA]	$\begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} m_0$	$Diag(1,-1,1)m_0$
[IB]	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_0$	$Diag(1,1,1)m_0$
[IC]	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} m_0$	$Diag(1,1,-1)m_0$
[IIA]	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} m_0$	$Diag(1,1,0)m_0$
[IIB]	$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} m_0$	$Diag(1,-1,0)m_0$
[III]	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} m_0$	$Diag(0,0,1)m_0$

Appendix B

Here we list the textures of the right-handed neutrino mass matrix M_{RR} along with the left-handed Majorana mass matrix m_{LL}^{I} generated through the canonical seesaw formula, Eq.(2), for three different models of neutrinos presented in Appendix A. The Dirac neutrino mass matrix is given in Eq.(7) where $m_f = m_\tau \tan\beta$ for case (i) and $m_f = m_t$ for case (ii), and $m_0 = m_f^2/(v_R)$. For normal hierarchical model the corresponding matrices are given in the main text. These are collected from Ref.[27] for ready reference.

Degenerate model(Type [IA]):

$$M_{RR} = \begin{pmatrix} -2\delta_2\lambda^{2m} & (\frac{1}{\sqrt{2}}+\delta_1)\lambda^{m+n} & (\frac{1}{\sqrt{2}}+\delta_1)\lambda^{m} \\ (\frac{1}{\sqrt{2}}+\delta_1)\lambda^{m+n} & (\frac{1}{2}+\delta_1-\delta_2)\lambda^{2n} & (-\frac{1}{2}+\delta_1-\delta_2)\lambda^{n} \\ (\frac{1}{\sqrt{2}}+\delta_1)\lambda^{m} & (-\frac{1}{2}+\delta_1-\delta_2)\lambda^{n} & (\frac{1}{2}+\delta_1-\delta_2) \end{pmatrix} v_R$$

$$-m_{LL}^{I} = \begin{pmatrix} (-2\delta_1+2\delta_2) & (\frac{1}{\sqrt{2}}-\delta_1) & (\frac{1}{\sqrt{2}}-\delta_1) \\ (\frac{1}{\sqrt{2}}-\delta_1) & (\frac{1}{2}+\delta_2) & (-\frac{1}{2}+\delta_2) \\ (\frac{1}{\sqrt{2}}-\delta_1) & (-\frac{1}{2}+\delta_2) & (\frac{1}{2}+\delta_2) \end{pmatrix} m_0$$

Degenerate model (Type [IB]):

$$M_{RR} = \begin{pmatrix} (1+2\delta_1+2\delta_2)\lambda^{2m} & \delta_1\lambda^{m+n} & \delta_1\lambda^{m} \\ \delta_1\lambda^{m+n} & (1+\delta_2)\lambda^{2n} & \delta_2\lambda^{n} \\ \delta_1\lambda^{m} & \delta_2\lambda^{n} & (1+\delta_2) \end{pmatrix} v_R$$

$$-m_{LL}^{I} = \begin{pmatrix} (1-2\delta_1-2\delta_2) & -\delta_1 & -\delta_1 \\ -\delta_1 & (1-\delta_2) & -\delta_2 \\ -\delta_1 & -\delta_2 & (1-\delta_2) \end{pmatrix} m_0$$

Degenerate model (Type [IC]):

$$M_{RR} = \begin{pmatrix} (1+2\delta_1+2\delta_2)\lambda^{2m} & \delta_1\lambda^{m+n} & \delta_1\lambda^{m} \\ \delta_1\lambda^{m+n} & \delta_2\lambda^{2n} & (1+\delta_2)\lambda^{n} \\ \delta_1\lambda^{m} & (1+\delta_2)\lambda^{n} & \delta_2 \end{pmatrix} v_R$$

$$-m_{LL}^{I} = \begin{pmatrix} (1-2\delta_1-2\delta_2) & -\delta_1 & -\delta_1 \\ -\delta_1 & -\delta_2 & (1-\delta_2) \\ -\delta_1 & (1-\delta_2) & -\delta_2 \end{pmatrix} m_0$$

Invereted hierarchical model(Type [IIA]):

$$M_{RR} = \begin{pmatrix} \eta(1+2\epsilon)\lambda^{2m} & \eta\epsilon\lambda^{m+n} & \eta\epsilon\lambda^{m} \\ \eta\epsilon\lambda^{m+n} & \frac{1}{2}\lambda^{2n} & -(\frac{1}{2}-\eta)\lambda^{n} \\ \eta\epsilon\lambda^{m} & -(\frac{1}{2}-\eta)\lambda^{n} & \frac{1}{2} \end{pmatrix} \frac{v_R}{\eta}$$

$$-m_{LL}^{I} = \begin{pmatrix} (1-2\epsilon) & -\epsilon & -\epsilon \\ -\epsilon & \frac{1}{2} & (\frac{1}{2}-\epsilon) \\ -\epsilon & (\frac{1}{2}-\epsilon) & \frac{1}{2} \end{pmatrix} m_0$$

Inverted hierarchical model(Type [IIB]):

$$M_{RR} = \begin{pmatrix} \lambda^{2m+7} & \lambda^{m+n+4} & \lambda^{m+4} \\ \lambda^{m+n+4} & \lambda^{2n} & -\lambda^{n} \\ \lambda^{m+4} & -\lambda^{n} & 1 \end{pmatrix} v_R$$

$$-m_{LL}^{I} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & -(\lambda^3-\lambda^4)/2 & -(\lambda^3+\lambda^4)/2 \\ 1 & -(\lambda^3+\lambda^4)/2 & -(\lambda^3-\lambda^4)/2 \end{pmatrix} m_0$$

The values of the parameters used are: Type IA: $\delta_1 = 0.0061875$, $\delta_2 = 0.0030625$, $m_0 = 0.4$eV; Type [IB] and [IC]: $\delta_1 = 3.6\times10^{-5}$, $\delta_2 = 3.9\times10^{-3}$, $m_0 = 0.4$eV; Type [IIA] and [IIB]: $\eta = 0.0001$, $\epsilon = 0.002$, $m_0 = 0.05$eV.

References

[1] For a recent review, see M.Maltoni, T.Schwetz, M.A.Tortola, J.W.F.Valle, **hep-ph/0405172**.

[2] A.Yu.Smirnov, **hep-ph/0402264**.

[3] G.Altarelli and F.Feruglio, **hep-ph/0405048**.

[4] Super-Kamiokande Collaboration, S.Fukuda et al., Phys. Lett. **B539**, 179 (2002), [**hep-ex/9807003**];
SNO Collaboration, S.N.Ahmed et al., Phys. Rev. Lett. **92**, 181301 (2004), [**nucl-ex/0309004**].

[5] Super-Kamiokande Collaboration, Y.Fukuda et al., Phys. Rev. Lett. **81**, 1562 (1998), [**hep-ex/9807003**].

[6] KamLAND Collaboration, K.Eguchi et al., Phys. Rev. Lett. **90**, 021802 (2003), [**hep-ex/0212021**].

[7] CHOOZ Collaboration, M.Apollonio et al., Phys. Lett. **B466**, 415 (1999), [**hep-ex/9907037**].

[8] K2K Collaboration, M.H.Ahn et al., Phys. Rev. Lett. **90**, 041801 (2003), [**hep-ex/0212007**].

[9] LSND Collaboration, A.Aguilar et al., Phys. Rev. **D64**, 112007 (2001), [**hep-ex/0104049**].

[10] BooNE Collaboration, E.D.Zimmerman, Nucl. Phys. Proc. Suppl. **123**, 267 (2003), **hep-ex/0211039**.

[11] WMAP Collaboration, C.L.Bennett et al., Astrophys. J. Suppl., **148** (2003) 1; D.N.Spergel et al., Astrophys. J. Suppl. **148**, 175 (2003); A.Kogut et al, Astrophys. J. Suppl., **148**, 161 (2003); G.Hinshaw et al., Astrophys. J. Suppl., **148**, 135 (2003); L.Verde et al., Astrophys. J. Suppl., **148**, 195 (2003); H.V.Peiris et al., Astrophys. J. Suppl., **148**, 213 (2003).

[12] S.Hannestad, JCAP **0305**, 004 (2003); O.Elgaroy and O.Lahav, JCAP **0304**, 004 (2003); S.Hannestad, **hep-ph/0310220**; S.Hannestad, G.Raffelt, **hep-ph/0312154**.

[13] The Heidelberg-Moscow Collaboration, H.V.Klapdor-Kleingrothaus et al., Eur. Phys. J, **A12**, 147 (2001); C.E.Aalseth et al., **hep-ex/0202026**.

[14] S.M.Bilenky, **hep-ph/0403245**.

[15] H.V.Klapdor-Kleingrothaus et al., Mod. Phys. Lett. **A37**, 2409 (2001); H.V.Klapdor-Kleingrothaus, A.Dietz, I.V.Krivoshena, Phys. Lett. **B586**, 198 (2004).

[16] G.Altarelli, F.Feruglio, Phys. Rep. **320**, 295 (1999), **hep-ph/9905536**.

[17] M.Gell-Mann, P.Ramond, and R.Slansky, in Supergravity, Proceedings of the Worshop, Stony Brook, New York, 1979, edited by P.van Nieueenhuizen and D.Freedman (North-Holland, Amsterdam, 1979); T.Yanagida, KEK Lectures, 1979 (unpublished); R.N.Mohapatra and G.Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).

[18] R.N.Mohapatra and G.Senjanovic, Phys. Rev. **D23**, 165 (1981); G. Lazarides, Q. Shafi, C. Wetterich, Nucl. Phys. **B181**, 287(1981); C.Watterich, Nucl. Phys. **B187**, 343 (1981).

[19] B.Brahmachari and R.N.Mohapatra, Phys. Rev. **D58**, 015001 (1998); Oleg Khasanov and Gilad Perez, Phys. Rev. **D65**, 053007 (2002); E.Ma, Phys. Rev. **D69**, 011301 (2004).

[20] B.Bajc, Senjanovic and F.Vissani, Phys. Rev. Lett. **90**, 051802 (2003), **hep-ph/0210207**; H.S.Goh, R.N.Mohapatra, S.P.Ng., Phys.Lett. **B57**, 215 (2003), **hep-ph/0303055**; Phys. Rev. **D68**, 115008 (2003), **hep-ph/0308197**.

[21] A.S.Joshipura, E.A.Paschos, W.Rodejohann, JHEP **0108**, 029 (2001), **hep-ph/0105175**; Nucl. Phys. **B611**, 227(2001), **hep-ph/0104228**.

[22] R.N.Mohapatra, **hep-ph/0402035**; **hep-ph/0306016**.

[23] B.Bajc, G.Senjanovic, F.Vissani, **hep-ph/0402140**.

[24] S.Antusch, S.F.King, **hep-ph/0402121**; S. Antusch, S. F. King, **hep-ph/0405093**.

[25] W.Rodejohann, **hep-ph/0403236**.

[26] T.Hambye, G.Senjanovic, Phys. Lett. **B582**, 73 (2004), **hep-ph/0307237**; P.O'Donnell, U.Sarkar, Phys. Rev. **D49**, 2118 (1994).

[27] N.Nimai Singh and M.Patgiri, IJMP **A17**, 3629 (2002); M.Patgiri and N.Nimai Singh, IJMP **A18**, 443 (2003).

[28] K.S.Babu, B.Dutta, R.N.Mohapatra, Phys. Rev. **D67**, 076006 (2003), **hep-ph/0211068**.

[29] For a discussion, see, I.Dorsner, S.M.Barr, Nucl. Phys. **B617**, 493 (2001); S.M.Barr, I.Dorsner, Nucl. Phys. **B585**, 79 (2000).

[30] E.Kh.Akhmedov, M.Frigerio, A.Yu.Smirnov, JHEP **0309**, 021(2003), **hep-ph/0305322**.

[31] D.Falcone, Phys. Lett. B479, 1 (2000), hep-ph/0204335.

[32] H.Murayama,**hep-ph/02010022**; A.de Gouvea, A.Friedland, H.Murayama, Phys. Lett. **B490**, 125 (2000).

[33] M.Patgiri and N.Nimai Singh, Phys. Lett. **B567**, 69 (2003).

[34] Carl H.Albright and S.M.Barr, **hep-ph/0404095**.

[35] K.S.Babu, S.M.Barr, Phys.Rev.Lett.**85**, 1170 (2000).

[36] H.Murayama and C.Pena-Garay, **hep-ph/0309114**.

[37] An incomplete list, Ernest Ma, Phys.Rev.**D69**,011301(2004),**hep-ph/0308092**; Utpal Sarkar, **hep-ph/0403276**; M.K.Parida, B.Purkayastha, C.R.Das, B.D.Cajee,**hep-ph/0210270**; B.Bajc, G.Senjanovic, F.Vissani, Phys.Rev.Lett.**90**,051802(2003); A Melfo, G.Senjanovic, Phys.Rev.**D68**,03501(2003).

[38] S. F. King, N. Nimai Singh, Nucl. Phys. **B596**, 81(2001).

[39] S. F. King, N. Nimai Singh, Nucl. Phys. **B591**, 3(2000).